Waves & Oscillations

In This Lecture…

- Mechanical Waves
- Transverse and longitudinal waves
- Wave equation
  - (mathematical description of a wave)
- Superposition (and decomposition)
- Interference

Mechanical Waves

- A Mechanical Wave travels with a material called a *medium*
- As wave travels through medium particles in the medium undergo displacement
- The speed of travel depends upon the mechanical properties of the medium

Mechanical Waves

- Examples a Mechanical Wave:

Types of Mechanical Waves

Mechanical Waves are either:

Transverse

OR

Longitudinal

Transverse Waves

- Displacement of medium is perpendicular (transverse to the direction of motion)
- Example: wave on a string

Transverse Waves

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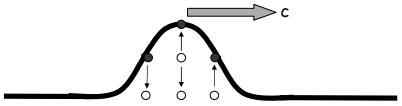
- Example: wave on a string



Longitudinal Waves

- Displacement of medium is perpendicular (transverse to the direction of motion
- Example: Air wave in a pipe

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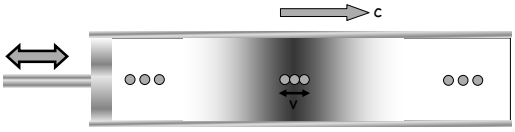


- Compression & Rarefaction

Mechanical Waves

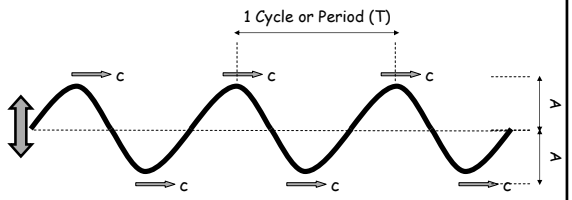
Three thing in common:

1. Disturbance travels with a definite speed through medium (propagation speed or wave speed)
2. Medium itself does not travel through space
3. Wave motion transports energy

Periodic Waves

- Sinusoidal motion
- Amplitude, A
- Frequency, f

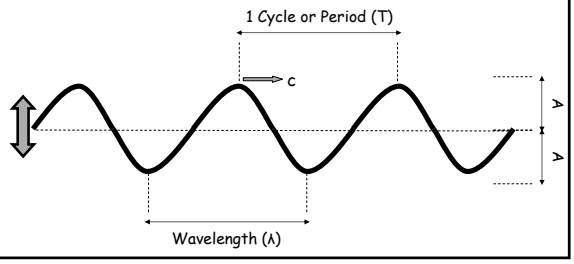
$$T = \frac{1}{f}$$



Periodic Waves

- Sinusoidal motion
- Wavelength, λ
- Wave Velocity, v

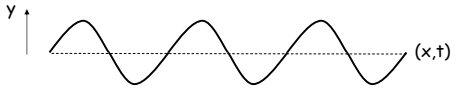
$$c = f\lambda$$



Wave Equation

- Mathematical Description of a wave
- A *function* that describes the amplitude of a wave at any given point or time

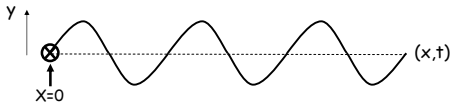
$$y = f(x,t)$$



Wave Equation $\omega = 2\pi f$

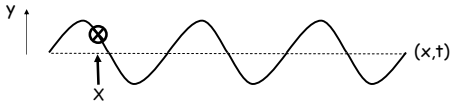
- The displacement at x = 0 (where motion originates) is given by:

$$y(x,t) = A\sin(\omega t) = A\sin(2\pi f t)$$



Wave Equation

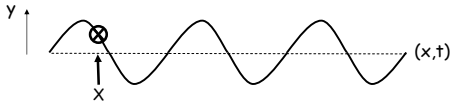
- Disturbance travels from x = 0 to x
- Time taken = distance/velocity = x/c



Wave Equation

- Motion of point x at time t is the same as the motion of point x=0 at the earlier time (t - x/c)
- Displacement at any point at time t can be found by replacing t with (t-x/c)

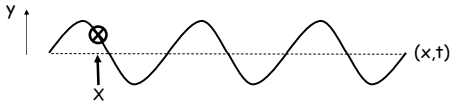
$$y(x,t) = A\sin\left(\omega\left(t - \frac{x}{v}\right)\right) = A\sin\left(2\pi f\left(t - \frac{x}{v}\right)\right)$$



Wave Equation $T = \frac{1}{f} \quad \lambda = \frac{v}{f}$

- Rewrite in terms of period (T) and wavelength (λ)

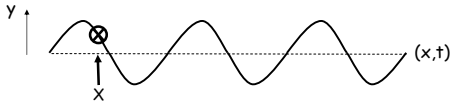
$$y(x,t) = A\sin\left(2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right)$$



Wave Equation $k = \frac{2\pi}{\lambda}$

- Another convenient form if we use *wave number* **k** $\omega = ck$

$$y(x,t) = A\sin(\omega t - kx)$$



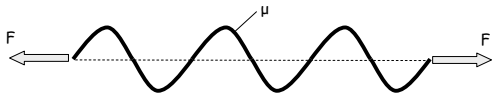
Speed of Transverse Wave

Wave Speed on a String

- Mechanical properties of string:
- Tension
  - Acts as resorting force
- Mass per unit length
  - Introduces inertia

$$c = \sqrt{\frac{F}{\mu}}$$

F = Tension in string
μ = Mass per unit length



Speed of Longitudinal Waves

Speed of longitudinal wave in a fluid filed pipe

- Mechanical properties of gas
- Compressibility
  - Acts as resorting force
- Density
  - Introduces inertia

$$c = \sqrt{\frac{B}{\rho}}$$

B= Bulk Modulus
ρ = Density

Speed of Longitudinal Waves

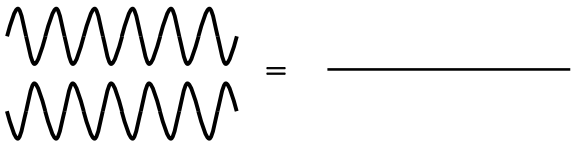
Speed of longitudinal wave in a solid rod

- Mechanical properties of rod
- Compressibility
  - Acts as resorting force
- Density
  - Introduces inertia

$$c = \sqrt{\frac{Y}{\rho}}$$

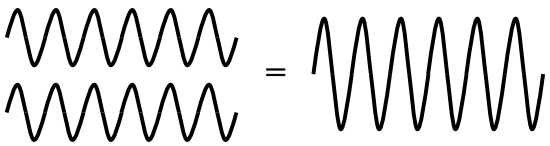
Y= Young's Modulus
μ = Density

Interference



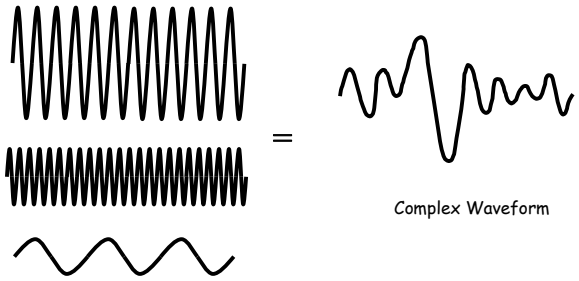
Destructive Interference

Interference



Constructive Interference

Superposition



Complex Waveform

Any waveform can be generated by varying amplitude, frequency & phase

Summary

- Mechanical Waves
- Transverse and longitudinal waves
- Wave equation
  - (mathematical description of a wave)
- Superposition (and decomposition)
- Interference

Practice Questions

PAM2011: Lecture 10 Problem Sheet Solutions

1. What is the wavelength of 1Mhz sound waves in air? (assume that the speed of sound in air is $344 ms^{-1}$)
2. The linear mass density of a string is $0.25 kgm^{-1}$. How much tension must be applied to produce a transverse wave with velocity of $10 ms^{-1}$?
3. The speed of sound in water at $20^{o}c$ is $344\ ms^{-1}$. Calculate the bulk module of water at this temperature. (Hint 1 litre of water weighs 1 Kg)
4. If wave is described by the following wave equation: $y=\sin(8\pi t - \pi x)$. Calculate it's velocity
5. Explain why the speed of sound in water is greater than that in air.